Finance 4000

Money and Capital Markets

Second class

Interest Rates

- Interest Rate — definition
  - A simple interest rate is the excess of the amount to be received in the future relative to the amount paid now
  - Formula
    - $$i_t = \frac{V_{t+1} - V_t}{V_t}$$
    - i_t is the interest rate
    - V_t is the value in period t — today
    - V_t is the value in period t+1 — next period or one period in the future
  - Given any two of the variables in this formula, you can figure out the other one
  - Examples

- Borrow \$100 and pay back \$105
  - V_t is \$100
  - V_{t+1} is \$105
  - i_t is .05 or 5 percent — (\$105-\$100)/\$100

- Borrow \$100 at 5 percent
  - V_t is \$100
  - i_t is 5 percent — or .05
  - V_{t+1} is \$105
  - often say “pay back \$100 and *interest payment* of \$5"
  - $V_{t+1} = (1 + i_t)V_t$

- If willing to pay \$105 a year from now and the interest rate is 5 percent, how much can you borrow?
  - V_{t+1} is \$105
  - i_t is 5 percent or .05
  - V_t is \$105
  - $$V_t = \frac{V_{t+1}}{(1+i_t)}$$

  - ads on tv
    - "If you have a structured settlement coming in a bunch of small payments, we will give you one large payment now."
    - How much?

  - Given some interest rate, you will get the *present value* of the future payments

- Present value

  - The present value of an amount to be received in the future is the value today of that payment.

  - A dollar in the future is not worth as much as a dollar today

- What if there is more than one payment?

- Compound interest

  - Suppose that we have $100 and we lend it at 5 percent per year for two years

  - How much should we get at the end of two years?

  - At the end of one year, we are owed $105
    - the original $100 plus 5 percent interest

  - We are willing to lend our funds for an additional year
    - from *t+1* to *t+2*
    - say 2000 to 2001

  - If the interest rate is 5 percent, then in 2001, we should get our $105 plus 5 percent interest
    - $105 + .05◦$105 = $105+$5.25 = $110.25
    - not $105 + $5
    - $.25 is interest on the interest from the first year

- The formula for this is determined the following way
  - $V_{t+1} = (1 + i_t)V_t$
  - and for the next period, this is
  - $V_{t+2} = (1 + i_{t+1})V_{t+1}$
  - For simplicity, suppose that the interest rate is constant at *i*
  - Then

$$V_{t+1} = (1 + i)V_t$$

$$V_{t+2} = (1 + i)V_{t+1}$$

  - And substituting the second into the first

$$V_{t+2} = (1 + i)[(1 + i)V_t] = (1 + i)^2 V_t$$

  -

$$V_{t+2} = (1 + i)^2 V_t$$

- The same proposition holds for this equation as for the simple one
  - Given any two of the variables in this formula, you can figure out the other one

- The formula for the present value of a payment two periods from today is

$$V_t = \frac{V_{t+2}}{(1+i)^2}$$

- Indicates how much willing to pay for a payment two years from now given an interest rate of *i*

- More than two payments in a "structured settlement"?

  - Just add up the present value of each of the payments

    - Let S_{t+1} be the structured settlement payment in period *t+1*
    - S_{t+2} be the payment in period *t+2*
    - and so forth

    - The present value of a structured settlement for 10 periods is

  - $$V_t = \frac{S_{t+1}}{(1+i)} + \frac{S_{t+2}}{(1+i)^2} + \frac{S_{t+3}}{(1+i)^3} + \ldots + \frac{S_{t+9}}{(1+i)^9} + \frac{S_{t+10}}{(1+i)^{10}}$$

    - Works for any number of years — 10, 4, or 99
    - Same payments every year, could get rid of subscripts on the payment — just use *S*

Financial Markets — Bonds

- Coupon bond
  - Price of bond — how much is paid for it
  - Maturity
    - The maturity of a bond is the time until the last payment
      - For example, a bond might have 20 years to maturity from today
  - Coupon payment
    - The coupon payment on a bond is a periodic payment made on a bond until maturity
      - A bond might pay $500 per year until maturity
  - Face value
    - The face value on a bond is a final payment
      - A bond might have a face value of $1000
  - What is interest rate on funds if hold bond to maturity?
  - The interest rate on funds if the bond is held to maturity is called the *yield to maturity*

- The yield to maturity is the interest rate that equates the present value of payments received from a credit market instrument with its value today

- Example — One year-bond
  - a price of $9000
  - a coupon payment of $500 per year
  - a face value payment of $10000
  - What is interest rate on funds?
  - i=($10000+$500-$9000)/$9000=.1666...
    - or 16.66... percent

- Two-year bond — two years to maturity

  - $$P_{b,t} = \frac{C_{t+1}}{(1+i)} + \frac{C_{t+2}}{(1+i)^2} + \frac{FV_{t+2}}{(1+i)^2}$$

  - Solve for yield to maturity *i*

Other kinds of bonds

- Treasury bills
  - Issued as discount securities
    - Only one payment
  - Term to maturity when issued of 90, 180 or 360 days
- STRIPS
  - Separately Traded Registered Interest and Principal Securities
  - Take a coupon bond and break it into single payments
  - For example, two year bond (often called a note)
    - two coupon payments of $1000
      - one in January 2000
      - one in January 2001
    - face value of $10,000 paid in January 2001
  - Take bond apart into 3 discount bonds
    - One paying $1000 in January 2000
    - One paying $1000 in January 2001
    - One paying $10,000 in January 2001
  - Why create a strip?

- For example, someone may have contract requiring that they pay $10,000 in January 2002

- Can buy bond and guarantee that they can make payment

- More generally, can put together any stream of receipts that is convenient

- Called zeroes

- State and local bonds
  - Typically not subject to federal income tax or state income tax if state of residence
- Private bonds
  - Higher risk than federal government
  - The higher the risk, the higher the yield

- Return from holding a bond
  - Really applies to any asset
  - Often called the *holding period return*
  - The holding-period return is the rate of return received over the period that the asset is held including any capital gains or losses on the asset
  - Let h be the holding-period return from holding a bond from t to $t+1$
    - The holding period return for one period is $h = \frac{P_{t+1} + C - P_t}{P_t}$
    - For one period, it is just
      - payment received
      - less payments made
      - initial payments made

- Example of holding-period return

  - Buy a 20 year bond today for $10,000

    - coupon payment of $1000 per year
    - face value of $10,000

    - yield to maturity is 10 percent

    - Suppose sell it a year from now for $9,000

    - The holding period return for one period is $h = \frac{P_{t+1} + C - P_t}{P_t}$

    - Means that holding period return is 0

      * $$\frac{\$9,000 + \$1,000 - \$10,000}{\$10,000}$$

  - If sold bond for $11,000 a year from now, *h* is

    - $$\frac{\$11,000 + \$1,000 - \$10,000}{\$10,000} = .20$$

      or 20 percent

- Bonds, Changes in Interest Rates and Risk

- Higher bond prices are associated with lower yields

  - The price of a bond equals the present value of payments

  - For a one-year bond $P_{b,t} = \frac{C_{t+1}}{(1 + i)} + \frac{FV_{t+1}}{(1 + i)}$

  - The bond contract specifies the coupon payments and the face value

  - Changes in the price are associated with changes in the interest rate (or yield to maturity)

    - Example — one-year bond

    - Coupon payment equal to zero

    - Face value equal to $1,000

    - If the price of the bond is $900, the interest rate is 11.11... percent

    - If the price of the bond is $950, the interest rate is about 5.26 percent

- If interest rates increase, the yield to maturity on existing bonds increases

- As a result, increases in interest rates lower prices of existing bonds

  - Bonds are subject to interest rate risk

  - The prices of longer term bonds decrease more when interest rates increase

  - Example

Prices of two bonds
Face value $10,000 and coupon payment $0

Interest Rate	Term to maturity (years) 1	2
5	9524	9070
10	9091	8264

Change in price -$433 -$806

- For a given increase in the interest rate, longer-term bond prices change more

- Empirically, longer-term bond prices are more variable

- Does that mean that one should hold short-term bonds for less risk?
    - Future short-term interest rates are unknown
    - Because of short-term interest rates are uncertain,

      Short-term securities are subject to *reinvestment risk*
    - Reinvestment risk is the risk that proceeds from securities will have to be reinvested at uncertain interest rates
- Long-term bonds are subject to interest-rate risk because prices change
- Short-term bonds are subject to interest-rate risk because of reinvestment risk
- Next time
    - Nominal and real interest rates
    - Duration as a measure of the maturity of a bond
    - Chapter 4 — portfolio choice