Finance 4000

Money and Capital Markets

Third class

- Duration and Interest-rate Risk of Bonds
    - Other things the same, the price of a bond with a longer term to maturity decreases more if interest rates increase
    - The price of a zero-coupon bond decreases more than the price of a coupon bond when the yields increase by the same amount
        - Example
    - Term to maturity does not capture some aspects of interest-rate riskiness
        - Example
    - A bond really consists of many intervening payments
        - Initial logic in terms of a loan
        - Variable payments such as zero coupon payment bond mean that such a structure need never be true
        - Term to maturity of each payment?
    - How measure term to maturity of combined payment streams and implications for risk of bonds?

- Duration

  - Term to maturity of each payment times fraction of present value of all payments at that term

  - Duration is the average lifetime of a stream of payments

- Equation

  - Let f_t equal the fraction of the present value of all payments at time *t*,
    - *t*=1, 2, 3, ..., *N* years in the future
    - *N* is term to maturity

  - $$Duration = \sum_{t=1}^{N} f_t t$$

  - f_t is from $$f_t = \frac{\frac{V_t}{(1+i)^t}}{\sum_{s=1}^{N} \frac{V_s}{(1+i)^s}}$$

- Duration and Interest Rate Risk

  - The longer the term to maturity, the longer the duration of a stream of payments

  - When the interest rate increases, the duration of a stream of payment decreases

  - The greater the coupon rate on a bond, the shorter the duration of a bond

  - Formula for effect of interest rate on price of a security

  - Let
    - $\%\Delta P_b$ be the *percentage* change in the price of a bond
    - Δi be the change in the interest rate

  - $\%\Delta P_b \approx -Duration\dfrac{\Delta i}{1+i}$

- The greater the duration of a security, the greater the decrease in price for a given increase in the interest rate

- Duration compared to term to maturity

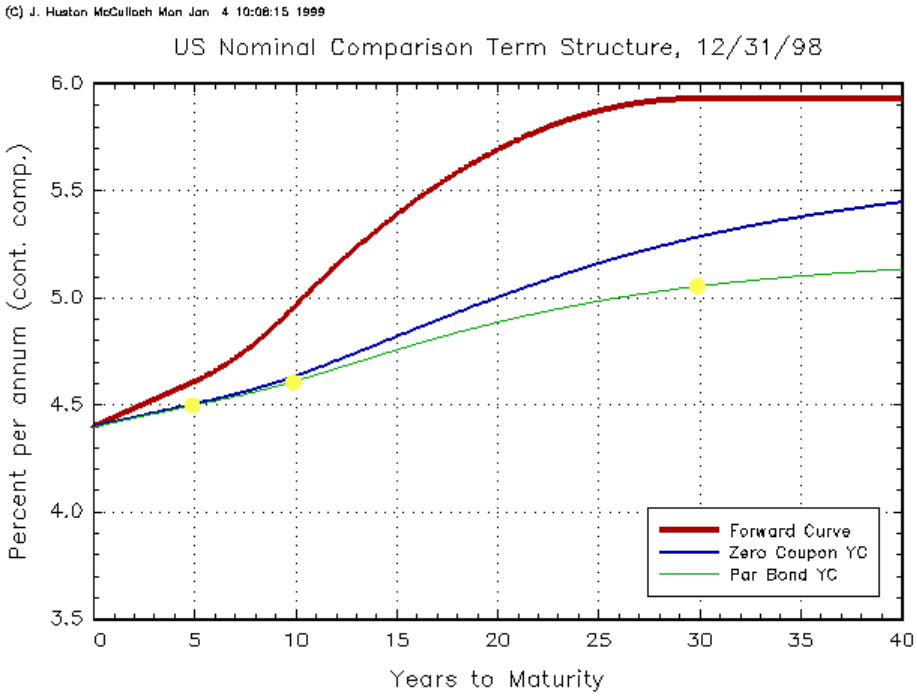
  - Both — longer associated with greater risk

  - Duration more effectively summarizes the relationship between the time to payment and interest-rate risk because it reflects the effect of intervening coupon payments

  - Can compute the duration of a portfolio of assets

    - Given same stream of payments, doesn't matter if coupon bonds or zeroes or whatever
      * Would not be true if used securities' terms to maturity

- Nominal and Real Interest Rates
    - A nominal interest rate is an interest rate in terms of dollars
    - A real interest rate is an interest rate in terms of commodities
    - Example
    - Treasury Inflation Protected Securities — TIPS
        - The number of dollars paid increases with the level of the Consumer Price Index
        - Example
        - Recent data


(C) J. Huston McCulloch Mon Jan 4 10:08:15 1999
US Nominal Comparison Term Structure, 12/31/98
Percent per annum (cont. comp.)
Years to Maturity
Forward Curve
Zero Coupon YC
Par Bond YC
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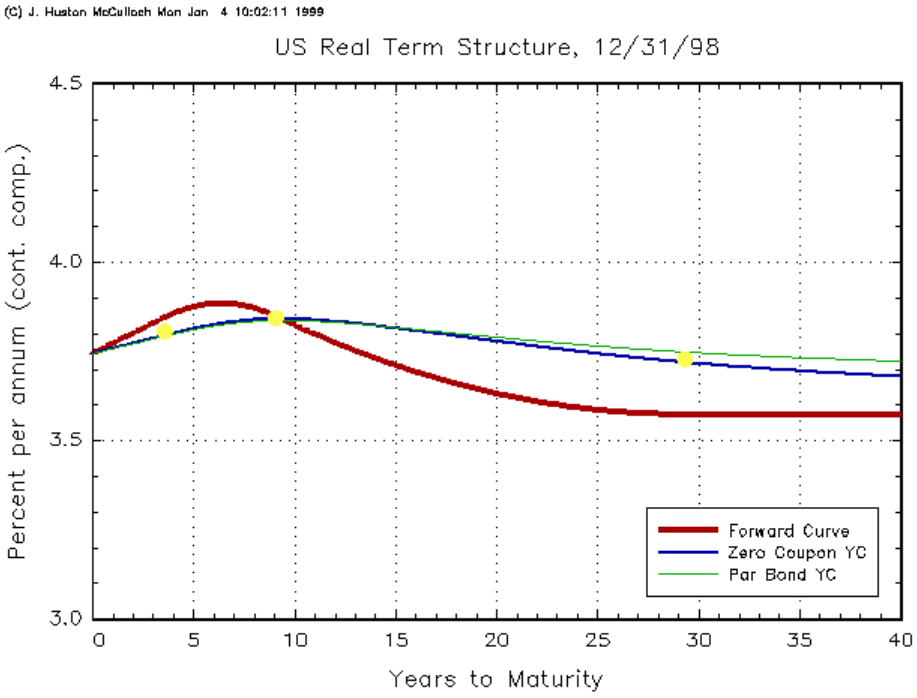
(C) J. Huston McCulloch Mon Jan 4 10:02:11 1999
US Real Term Structure, 12/31/98
Percent per annum (cont. comp.)
Years to Maturity
Forward Curve
Zero Coupon YC
Par Bond YC
4.5
4.0
3.5
3.0
0
5
10
15
20
25
30
35
40

- Portfolio Choice
  - An asset is a possession having value
    - due to anticipated payments or higher value for a security
  - At a general level, the demand for securities is straightforward
  - Demand for securities depends on
    - Wealth
    - Expected return on the security relative to other securities
    - Risk of the security
      - a chance or possibility of danger, loss, injury or other adverse consequences
    - Liquidity
      - the ability to be cheaply and quickly converted into cash
    - Not as informative as it could be because it's not very specific

- Theories of the demand for securities
  - Capital Asset Pricing Model (CAPM)
    - Investors are risk averse and must be compensated for bearing risk
    - The expected return on securities will tend to be higher, the higher the co-movement of their expected returns with market returns

  - Arbitrage Pricing Theory (APT)
    - Investors are risk averse and must be compensated for bearing risk
    - The expected return on securities will tend to be higher, the higher the co-movement of their expected returns with factors that affect all securities

- Risk Generally Measured by the Variance of the Expected Return

  - Expected return

    Two possible outcomes

    - $Er = p_1 r_1 + p_2 r_2$
      - where
      - p_1 is the probability of event 1 with return r_1
      - p_2 is the probability of event 2 with return r_2
      - $p_1+p_2=1$
    - Average return expected in the future

  - Variance
    - $\sigma^2 = p_1(r_1 - Er)^2 + p_2(r_2 - Er)^2$

  - Standard Deviation

    - $\sigma = \sqrt{\sigma^2}$
    - Same units as expected return
    - A rough rule of thumb for securities is that an expected return more than one standard deviation from the expected return happens about 1/3 of the time

  - When is the standard deviation a good measure of risk? When not?

- How Reduce Risk?
  - Modern Portfolio Theory: Diversify
  - Example
  - Lessons
    - Diversification can reduce risk if outcomes independent
    - Diversification can reduce risk more if bad outcomes are negatively related
      * With symmetry, bad outcomes negatively related implies good outcomes negatively related
  - International Diversification — handout
  - The *efficient portfolio frontier* is the set of the most attractive combinations of the expected return and standard deviation of the expected return on the securities
- As add more securities, the risk due to holding the securities decreases
- Decrease in risk is limited by the variance common to the securities

- For United States, get close to market risk with just 20 securities

    - Market risk for stocks is the variance of expected return on all stocks weighted by their relative values

- The expected return will include compensation for systematic (non-diversifiable) risk but not for nonsystematic, or idiosyncratic, risk

    - Can diversify away nonsystematic risk

    - Results in nonsystematic risk being unpriced

- Capital Asset Pricing Model

  - Systematic risk consists of changes in expected return associated with changes in the market expected return

  - Suppose there is a linear relationship between the expected return on individual assets and the market return

    - $Er_i = \alpha_i + \beta_i r_m + \varepsilon_i$

    - Systematic variance in expected return can be represented by βr_m

    - Unsystematic variance in expected return is represented by ε_i

  - Suppose that there is a risk-free asset with a risk-free expected rate of return

  - Combinations of the risk-free asset and diversified portfolios are possible

  - The efficient portfolio frontier shows the set of most preferred portfolios of risky securities

- If there is a zero-variance security, then the efficient portfolio frontier including the risk-free asset is a straight line between expected return on the portfolio and the standard deviation of the portfolio

- Knowing what will happen requires specifying more about investors' preferences and equilibrium

- If everyone is identical, they must hold the market portfolio

- In equilibrium, $Er_i = Er_f + \beta(Er_m - Er_f)$

  - β is the marginal contribution of security *i* to the risk in the portfolio

  - Er_m-Er_f is called the market price of risk because it is the expected marginal return from holding the market portfolio instead of the risk-free asset

- What's wrong with the Capital Asset Pricing Model?

- Arbitrage Pricing Theory

  - Basically examines nondiversifiable risk in terms of factors that affect the expected return on each security

  - May prove to be more stable or informative empirically