First Exam
Economics 8050
Macroeconomics II
Fall 2000

1. In an overlapping generations economy, suppose that household *h* in generation *t* has the utility function $U^h = c_t^h(t)c_t^h(t+1)$ where $c_t^h(s)$ is consumption by member *h* of generation *t* in period *s*=*t*, *t*+1. Suppose that the number of people in each generation is constant at an even number *N*.

Suppose that the lower half of the households (1,2,...,2/*N*) in generation *t* receive an endowment equal to 2ω in period *t* and 0 in period *t*+1. Suppose that the upper half of the households ((2/*N*)+1,(2/*N*)+2,...*N*) in generation *t* receive an endowment equal to 0 in period *t* and an endowment equal to 2ω in period *t*+1.

Suppose that the government collects lump-sum taxes, makes lump-sum transfer payments and issues no debt. Derive equations characterizing the government's budget, the household saving functions and the aggregate saving function in each period. Also derive the function for the equilibrium interest rate in each period.

a. Suppose that the government has been collecting no taxes and that the government starts a transfer program of taking from the rich and giving to the poor. When people are "young", (i.e. generation *t* in *t*), the government collects taxes of ω from households with a positive endowment and transfers the proceeds to the households with no endowment. When people are "old", (i.e. generation *t* in *t*+1), the government collects taxes of ω from the households with a positive endowment and transfers the proceeds to the households with no endowment. Show the effect on the equilibrium consumption of the rich and the poor, the equilibrium interest rate and equilibrium borrowing and lending.

b. Outline how you think your answer in a. would change if the taxes were not lump sum, instead being taxes on labor income that has an opportunity cost of valuable leisure. (This question requires no algebra.)

2. It sometimes is noted that everyone does not expect the same thing, and it cannot literally be true that people's expectations of a variable, say x_{t+1} are given by $E[x_{t+1} \mid \Omega_t]$, where Ω_t is the common information set available to people. Consider two possibilities and their implications for tests of rational expectations. For the purposes of this question, assume that people have rational expectations in the sense that their expectations conditional on the information actually available to them are the same as mathematical expectations.

a. Suppose that people have access to different information, so that the household *i* has expectation

$$x_{i,t+1}^e = E[x_{t+1} \mid \Omega_{i,t}]$$

where $\Omega_{i,t}$ is the information set available to household *i* and these information sets are not all the same. Suppose that a researcher proposes to test whether "expectations are rational" by the ordinary least squares regression

(1) $$x_{t+1} = \alpha + \beta x^{e}_{i,t+1} + \varepsilon_{i,t}, \qquad i = 1,...,N \quad t = 1,...,T$$

where α and β are parameters and $\varepsilon_{i,t}$ is the error term. Show whether or not the usual maintained hypotheses of $\beta = 1$ and $\alpha = 0$ hold.

b. Suppose that individual households' expectations are not due to different information but are due to noise added to the mathematical expectation, so household *i*'s expectation is given by

$$x^{e}_{i,t+1} = E[x_{t+1} \mid \Omega_t] + \eta_{i,t+1}$$

where the error term $\eta_{i,t+1}$ has zero mean and constant variance, is serially uncorrelated, and the correlation of $\eta_{i,t+1}$ and $E[x_{t+1} \mid \Omega_t]$ is zero. Suppose that a researcher proposes to test whether "expectations are rational" by the ordinary least squares regression (1). Show whether or not the usual maintained hypotheses of $\beta = 1$ and $\alpha = 0$ hold.